

# Redefining Information and Efficiency to Understand Technical Analysis\*

Sherwin Lott<sup>†</sup>

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## Abstract

Encrypted data is hardly informative without a key, yet the efficient market hypothesis suggests that all complex patterns and interactions between prices are accounted for. Economics, unlike probability, must recognize costs and therefore distinguish between observing pieces of information and analyzing their many interactions. (1) We generalize  $\sigma$ -fields to families of events and define information more broadly as knowledge about optimization solutions. (2) This provides a new framework for efficiency hypotheses and theorems. (3) We illustrate how complex patterns arise from “variably diffuse information” that only technical analysts can aggregate indirectly, changing the informational behavior of prices.

**Keywords:** Market Efficiency, Information Aggregation, Price Informativeness, Bounded Rationality, Technical Analysis.

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<sup>†</sup>University of Pennsylvania Department of Economics, [lotts@sas.upenn.edu](mailto:lotts@sas.upenn.edu).

# 1 Introduction

The efficient market hypothesis (EMH), as established by Fama (1970, 1991), precludes any rationalization of technical analysis—despite being the dominant form of trading behavior at short horizons (Allen and Taylor 1990; Cheung and Chinn 2001; Gehrig and Menkhoff 2006; Menkhoff 2010). Until such a juxtaposition between theory and practice is resolved, economists cannot be confident in their understanding of markets, from policy and trading advice to the informational behavior of prices. The question is what information does public data contain that only technical analysts can effectively aggregate? Our answer is to (1) redefine information and (2) the EMH so that private information can be learned from public data and (3) illustrate why technical analysts are needed to aggregate “variably diffuse information.”

First, the way information is defined in economics via probability theory makes no distinction between observing data and running every possible analysis. That is, there is no direct way to condition on pieces of information without also conditioning on all their exponentially many interactions. While useful for probabilistic foundations, this narrow definition fails to recognize that analyzing interactions is costly. Traders might reasonably know pieces of information without fully understanding their interactions. Rather than framing this as just bounded rationality, information should be defined broadly to include analysis.

Without changing any rules of probability spaces, we generalize how information is defined via  $\sigma$ -fields to *families* of events.<sup>1</sup> The point is that families do not have to be closed under intersection, allowing pieces of information to be conditioned on without their many interactions. While there is no obvious way to make this work probabilistically, it readily follows when information is defined as knowledge about optimization solutions. Specifically, conditioning on a family means knowing how to optimize over all functions at least as *coarse*. In finite sample spaces, coarseness naturally extends

1. See Appendix A for a review of  $\sigma$ -fields and families using examples in Section 2.

and is consistent with the probabilistic definition of conditioning on  $\sigma$ -fields.

By viewing analysis as a form of information, it becomes apparent just how strong even the weakest version of the EMH really is. For markets to have no *excess return predictability*, all the interactions contained in troves of public time series data must effectively be common knowledge. Given the complexity of markets, there is little reason to expect this approximation holds *a priori* even with significant incentives. Indeed, the sheer number of traders analyzing public data to predict excess returns cannot be rationalized if interactions are so well understood.

Our basic version of the efficient market hypothesis (EMH\*) then says that prices are as they would be if all private analyses of public data were common knowledge. This mathematically formalizes what information prices do not reflect in a way that could only be verbalized previously. Of course, analyzing data is costly, so prices only partially reflect public data to the extent traders have been incentivized to analyze it. As analytical methods improve over time, it is not particularly surprising under EMH\* how many profitable trading strategies are found retroactively in the finance literature that were already obsolete (Park and Irwin 2007).

While efficiency hypotheses are conceptually useful, there are many reasons why markets might not fully aggregate private analyses. For example, traders must still earn information rents as in Grossman and Stiglitz (1980). Rather, like the welfare theorems, sufficient conditions for hypotheses provide a starting point to understand what can go wrong, and digging into their nuances may deliver a flourishing theory. The first step is to appropriately define information, which has implications across economics, but most directly to the EMH and information aggregation. After generalizing  $\sigma$ -fields to families and deriving efficiency results, the question becomes what is the broadest way to define information?

Economics is about decision making, and the fundamental basis for it is knowledge about optimization solutions. This sort of knowledge might be familiar on a practical level because it includes everything naturally done when computation is limited, which

is entirely the point—analysis is costly and constitutes its own form of information. We develop a mathematical structure around induction to describe how someone might be knowledgeable about optimization solutions and then make inferences. This gives a new perspective on behavioral questions and game theory, but it is likely too abstract and unconstrained for direct application.

Having belabored ways of defining analysis, we turn to why technical analysts are prevalent by explaining their unique informational role. How do patterns arise that other traders, such as value investors and liquidity providers, cannot effectively model? Our answer is that variation over information structures is hard to observe but affects price through its interaction with demand. Patterns then result between associated variables and price that seem economically spurious and cannot be readily derived.

More specifically, for any given asset and point in time, it is hard to know how many traders are informed about what. Even in the simplest model where traders draw from the same univariate signal distribution, there will inevitably be aggregate variation in the number of traders and their distributional moments. These interact with demand curvature and generate pricing errors to the extent traders do not account for them. While directly estimating such a latent process might be too difficult, theoretical models can at least illustrate how prices behave and what patterns to look for.

For example, Miller (1977) shows that unexpectedly disperse opinions cause higher prices through the winner's curse. Their effect is magnified by the relative wealth of potential traders to market capitalization, so economic policies and variables that increase this ratio can reduce price informativeness. This comparative static is notably the reverse of noise trading results stemming from Grossman (1976). Ultimately, many different sources of noise cumulatively determine the behavior of prices.

Only by modeling these sources of noise can traders better figure out which are significant in what markets and how to better account for them. Even if traders do not use well specified parametric models, theory can still guide what profitable strategies to look for. Dispersion and the winner's curse suggest that interactions between related

variables need to be accounted for, such as: analyst variation, short interest, liquidity, and market depth. To what extent these interactions have been accounted for is then a question in the efficiency literature (Diether, Malloy, and Scherbina 2002; Johnson 2004; Sadka and Scherbina 2007; Moeller, Schlingemann, and Stulz 2007).

More generally, we illustrate how random effects and distortions over information structures cause pricing errors with many different implications for trading and price informativeness. The complex interactions that arise in just our toy model belie how intractable they are to parametrically model in real markets. Technical analysts who broadly decrypt financial data may then have a comparative advantage finding these seemingly economically spurious interactions over conventional traders.

The remainder of this paper is organized as follows. Section 2 presents the EMH and how it is limited by information sets. Section 3 extends conditioning to families of events and gives the corresponding efficient market hypothesis (EMH\*), for which Section 4 develops sufficient conditions. Section 5 illustrates how “variably diffuse information” can generate the kind of short-term economically spurious patterns that technical analysts look for, and Section 6 concludes.

## 2 The Efficient Market Hypothesis (EMH)

While many different versions of the EMH have been proposed, they all follow the same common formulation given by Jensen (1978):

A market is efficient with respect to information set  $\theta_t$  if it is impossible to make economic profits by trading on the basis of information set  $\theta_t$ .

Each version of the EMH then corresponds to a specific choice of information set  $\theta_t$ , which generally includes all past security prices—if not harder to access public data or even private information.

## 2.1 Specifying a Joint Hypothesis

The EMH is notably incomplete without a joint hypothesis that specifies exactly how information should translate into prices. Whether or not economic profits can be made by a given trader on the basis of information set  $\theta_t$  depends on their preferences and priors. Implicitly, everything is being defined here with respect to some representative trader, so what are *reasonable* priors for them to have?

Given some canonical preferences, if it is to be said prices fully reflect information set  $\theta_t$ , then we argue traders must be able to learn any sufficiently repeated pattern contained within. Patterns at shorter horizons with higher frequency will naturally be easier to demonstrate. For example, if there were a large Monday effect, traders with information sets containing all past security prices should know of it. Whereas, say the long-run effect of macroeconomic policies on fundamentals might require a much longer history to discern. This ability to learn patterns is a constraint on how degenerate and misspecified priors can be.

Unless we specify that knowing information entails having fully introspected the relevant priors, then knowledge itself can be an arbitrarily weak notion. At a minimum, we argue that fully introspected priors should not preclude the ability to learn patterns. While there remains a gray area as to what exactly constitutes a pattern or how much information a trader would need to learn one, it still conceptually narrows down what joint hypotheses pair meaningfully with the EMH.

## 2.2 The Problem with Information Sets

Having specified the relevant set of joint hypotheses, we can now try to explain the disconnect between trading behavior in practice and the EMH in theory. Our critique is that the very concept of information sets is too narrow: having pieces of information necessarily means being able to condition on all their interactions. There exists a whole possible spectrum of information by which any level of interactions could be known.

We illustrate this limitation with Example 1 and Example 2.

**Example 1 (Separability)** *Consider the following two random variables  $X_1 \in \{0, 1\}$  and  $X_2 \in \{0, 1, 2\}$  in the corresponding probability space  $(\Omega, 2^\Omega, \mathbb{P})$ .*

		$X_2$			
		$\Omega$	0	1	2
$X_1$	0	$\omega_1$	$\omega_2$	$\omega_3$	
	1	$\omega_4$	$\omega_5$	$\omega_6$	

Table 1: Separability example.

In probability theory, information is defined with respect to  $\sigma$ -fields which describe what events can be conditioned on. (See Appendix A for a basic review of  $\sigma$ -fields in the context of these examples.) Notably,  $\sigma$ -fields are families of events that are closed under countable intersection and complementation. Observing an information set, e.g. a collection of random variables or family of events, necessarily means being able to condition on all of the events in the  $\sigma$ -field they generate. However, these closure properties make it so that pieces of information cannot be known without their interactions.

In the probability space given by Example 1, knowing  $X_1$  and  $X_2$  necessarily means being able to condition on all their interactions since  $\sigma(X_1, X_2) = 2^\Omega$  is the total  $\sigma$ -field. Formally, let event  $E_i^j$  denote:

$$E_i^j = \{X_i = j\}$$

Note that any any singleton event  $\{\omega_i\}$  in the sample space can be expressed as the intersection of (two) such events, e.g.  $E_1^0 \cap E_2^0 = \{\omega_1\}$ . By closure under intersections, there is no way someone could know the events corresponding to  $X_1$  ( $E_1^0$  and  $E_1^1$ ) and  $X_2$  ( $E_2^0$ ,  $E_2^1$ , and  $E_2^2$ ) without also knowing all their interactions ( $2^\Omega$ ).

Nor can separability be expressed via sufficient statistics because the generated sub  $\sigma$ -field would still need to be closed under intersection. One solution is to set

up the information structure so that nothing can be learned from conditioning on interactions. However, this abstracts the content of knowledge away from the observed data to specifics of the information structure, e.g. priors and global games. This does not lend itself to an EMH that specifies what parts of publicly available financial data is and is not accounted for in market prices.

We want a general mathematical system where  $X_1$  and  $X_2$  can be known separately without their interactions. This means being able to condition on each of three and two events corresponding to  $X_1$  and  $X_2$  but not each of the six realizations in the sample space. Traders would effectively have five degrees of freedom instead of six. Economics, unlike probability theory, has to do with what people know rather than what could possibly be inferred. Rather than requiring information be closed under intersection, we suggest traders may know families of events.

While it would be trivial for traders to interact  $X_1$  and  $X_2$  in Example 1, the curse of dimensionality in large financial time series makes this distinction much more meaningful. If seemingly spurious interactions are prevalent in security prices, as trading behavior and Section 6 suggest, then there is little reason to expect each interaction would be known by some trader let alone by all. Traders may then be able to derive a great deal of private information from analyzing public data. The point is not just that analyzing data is costly, but rather that information sets are not specific enough to distinguish between what analyses are public, private, and unknown.

**Example 2 (Curse of Dimensionality)** *Consider the collection of random variables  $\{X_t^n\}_{t=1, n=1}^{T, N}$  that can each take on  $M$  different possible values:  $X_t^n \in \{x_{t,1}^n, \dots, x_{t,M}^n\}$ . The size of the sample space,  $|\Omega| = M^{TN}$ , is exponential in  $T$  and  $N$ .*

The point of Example 2 is merely that interactions can grow exponentially with the amount of data, i.e. the curse of dimensionality. It then becomes critical to distinguish between separately knowing pieces of information from their interactions, as the former grows linearly. Otherwise, there is a bait-and-switch in our intuitions about the EMH.

		$X_{t+1}^n$			
$\Omega$		0	1	$\dots$	$x_{t+1,m}^n$
$X_t^n$	0	$\omega_{i_1}$	$\omega_{i_2}$	$\dots$	$\omega_{i_m}$
	1	$\omega_{i_{m+1}}$	$\omega_{i_{m+2}}$	$\dots$	$\omega_{i_{2m}}$
	$\vdots$	$\vdots$	$\vdots$	$\ddots$	$\vdots$
$x_{t,m}^n$	$\omega_{i_{m^2-m+1}}$	$\omega_{i_{m^2-m+2}}$	$\dots$	$\omega_{i_{m^2}}$	

Table 2: Cross section of table with T x N x M dimensions.

Any security price at a given point in time seems to clearly be public information and ought to be included by information set  $\theta_t$  in the EMH. However, the  $\sigma$ -field generated by all past security prices taken together becomes large enough that there's no reason to expect *a priori* it will be fully accounted for.

Standard versions of the EMH are still conceptually and empirically useful for many reasons given by Fama (1991). However, we question here the underlying intuition for the EMH being a good approximation of markets. There is little reason to expect that all the information contained in security prices is approximately common knowledge and therefore reflected by prices. To the extent the EMH oversimplifies, it misleads traders about what strategies to look for.

### 3 Conditioning on Families of Events

The way information is defined via  $\sigma$ -fields makes no explicit distinction between simply observing pieces of information and analyzing their many interactions. This leads to a fallacious intuition where if each piece of information is publicly known, then the  $\sigma$ -field generated by their collection must be as well. Separability and curse of dimensionality are examples of what difficulties get incidentally shoved away. Our solution is to define information and conditioning with respect to families of events more generally, leading to alternate versions of the EMH.

### 3.1 Redefining Information

Let  $(\Omega, \mathcal{F}, \mathbb{P})$  denote the underlying probability space, then knowing a family  $\mathcal{G} \subset \mathcal{F}$  is defined as being able to optimize over all functions with coarseness  $\mathcal{G}$ . A function has coarseness  $\mathcal{G}$  if it can be written as the sum of indicator functions over events in  $\mathcal{G}$ , which Definition 1 formalizes. To avoid complications with infinite sums, we assume the sample space is finite, which could be as large as any bounded digitized data set.

Our following notion of *coarseness* extends the concept of measurability from  $\sigma$ -fields to families of events more generally.

**Definition 1 (Coarseness)** *The set  $F_{\mathcal{G}}$  of all functions with (finite) coarseness  $\mathcal{G}$  is:*

$$F_{\mathcal{G}} \equiv \left\{ f(\omega) \mid f(\omega) = \sum_{E \in \mathcal{G}} c_E \mathbb{1}_{\{\omega \in E\}} \text{ for some } \{c_E\}_{E \in \mathcal{G}} \text{ constants with respect to } \Omega \right\}$$

Analogously,  $F_{\mathcal{G}}(y)$  is the set of all such functions where  $c_E$  can be a function of  $y$ .

Conditioning on a family  $\mathcal{G}$  means being able to optimize over all functions with coarseness  $\mathcal{G}$ . Definition 2 characterizes a solution as being that which optimizes the unconditional expectation.

**Definition 2 (Argmax)** *The set of solutions to a maximization problem conditional on a family  $\mathcal{G}$  is given by:*

$$\begin{aligned} \arg \max_{x \in X} u(x \mid \mathcal{G}) &\equiv \arg \max_{\hat{x} \in F_{\mathcal{G}}, \hat{x}(\Omega) \subseteq X} \mathbb{E}[u(\hat{x}(\omega))] \\ &\subseteq \{x^* : \Omega \rightarrow X\} \end{aligned}$$

Where the function  $u$  depends on the imperfectly observed realization  $\omega$ .

Having defined  $\arg \max$ , all of the corresponding definitions for  $\max/\min$  and  $\sup/\inf$  readily follow.

Notably, if  $\mathcal{G}$  is a  $\sigma$ -field, then conditioning turns into its standard probabilistic definition. The reason being that the smallest events in  $\mathcal{G}$  generate a partition over  $\Omega$

that can be separately optimized over. Thus, if  $\mathcal{G}$  is a  $\sigma$ -field, then for any  $\omega \in \Omega$ , any optimal solution  $x^*(\cdot | \mathcal{G})$  is necessarily such that:

$$x^*(\omega | \mathcal{G}) \in \arg \max_{x \in X} \mathbb{E}[u(x) | E_\omega], \text{ where } E_\omega = \bigcap_{E \in \mathcal{G}, \omega \in E} E$$

**Property 1** *Conditioning on families (Definition 2) is consistent with the standard probabilistic definition for conditioning on  $\sigma$ -fields and information sets.*

### 3.2 Generalizing the EMH

Having generalized the concept of information, we can now consider a much broader class of efficient market hypotheses than Jensen (1978). Each hypothesis here may correspond to a family  $\mathcal{G}_t$  rather than an information set  $\theta_t$ :

A market is efficient with respect to family  $\mathcal{G}_t$  if it is impossible to make economic profits by trading on the basis of family  $\mathcal{G}_t$ .

The critical distinction being that a family  $\mathcal{G}_t$  may include pieces of information without all of their interactions. What interactions have been analyzed can readily be demarcated using families.

Specifically, if  $\mathcal{G}_t$  is the union of all private analyses,  $\mathcal{G}_t = \cup_i \mathcal{G}_t^i$  (where traders are indexed by  $i$ ), then the EMH becomes:

**Definition 3 (EMH\*)** *A market is informationally efficient if prices need not differ from the counterfactual where all private analyses of public data are common knowledge.*

To avoid the notion of economic profits because they depend on individual preferences, efficiency is defined here in terms of equilibria.

It should be emphasized that EMH\* is no empirical substitute for standard versions of the EMH that provide useful null hypotheses to test against. Rather, it challenges the underlying intuition for markets approximately aggregating all of the information contained in public data. The vastness of security price data combined with the curse of

dimensionality leaves a potentially large gap between the information contained within and the union of all private analyses.

## 4 Sustaining Efficient Demand

The goal here is to find sufficient conditions under which an aggregate demand function  $D^*$  that leads to EMH\* can be sustained in equilibrium. Our model is set up in such a way that the only question is whether traders with the relevant information have enough liquidity to construct  $D^*$ . If they can construct  $D^*$ , then because it is efficient, their preferences will fall into place such that it can be sustained in equilibrium.

Specifically, we consider a continuum of traders who are risk-neutral and liquidity constrained. Traders choose demand  $x_1, \dots, x_K$  in dollars for  $K$  different risky assets and save the rest of their wealth in a risk-free storage technology. Traders are able to use their liquidity to short trade an asset and receive a rate of return equal to that of the storage technology minus the asset. The realized total value of these assets  $v_1, \dots, v_K$  are positive random variables measurable with respect to the underlying finite probability space  $(\Omega, 2^\Omega, \mathbb{P})$ .

Given an aggregate demand function  $D^*$ , since the probability space is finite, there are finitely many types of traders  $i \in \{1, \dots, I\}$  characterized by their information, i.e. family of events  $\mathcal{G}_i \subseteq 2^\Omega$  (which may depend on the prices that result from  $D^*$ ). Because traders are small and risk neutral, if  $D^*$  has coarseness  $\mathcal{G}^* = \cup_i \mathcal{G}_i$  and leads to EMH\*, then a trader of type  $i$  is indifferent between all their possible trading strategies if  $\mathcal{G}_i \subseteq \mathcal{G}^*$ . Thus, we just need to show that the traders are capable together of constructing  $D^*$  given each information type has some liquidity  $l_i$ .

Let aggregate demand  $D^*$  have coarseness  $\mathcal{G}^*$ , then it can be decomposed into:

$$D^*(\omega) = \sum_{E \in \mathcal{G}^*} c_E^* \mathbb{1}_{\{\omega \in E\}}$$

The coefficients  $|c_E^*|$  give an upper bound on how much liquidity needs to be “reserved” towards the event if it happens. That is, we need to be able to match trader types to

events in  $\mathcal{G}^*$  in sufficient measure to construct all the coefficients  $c_E^*$ .

This turns into the following bipartite matching problem where the inflow to trader type  $i$  is their liquidity  $l_i$ , and the required outflow from each event is  $c_E^*$ . Sufficient conditions for there being a solution to this matching problem are given by Hall's marriage theorem.

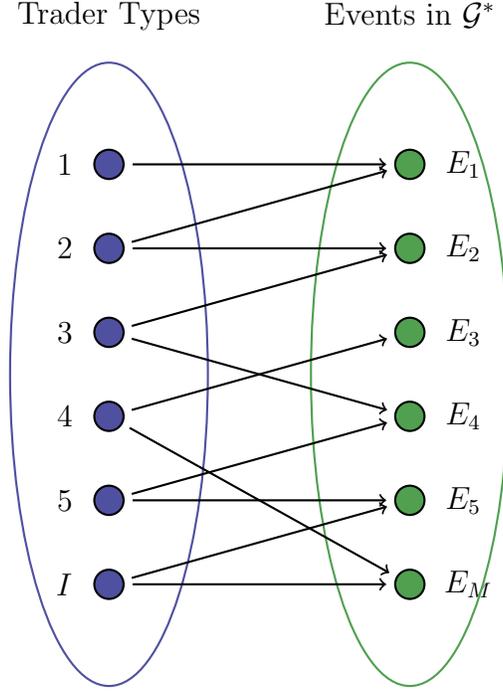


Figure 1: Bipartite flow matching problem with *events* in  $\mathcal{G}^*$ .

**Result 1** *An aggregate demand function  $D^*$  with coarseness  $\mathcal{G}^* = \cup_i \mathcal{G}_i$  that leads to equal returns across assets (conditional on  $\mathcal{G}^*$ ) can be supported in equilibrium if for all families  $\hat{\mathcal{G}} \subseteq \mathcal{G}^*$ :*

$$\sum_{E \in \hat{\mathcal{G}}} |c_E^*| \leq \sum_{i: \mathcal{G}_i \cap \hat{\mathcal{G}} \neq \emptyset} l_i$$

Where  $c_E^*$  corresponds to the constant multiplying  $\mathbb{1}_{\{\omega \in E\}}$  in the decomposition of  $D^*$ .

**Proof.** Hall's marriage theorem and the triangle inequality (see Appendix B). ■

Intuitively, these sufficient conditions are lower bounds for the liquidity different subsets of traders need to have. Enough traders with the right information are needed to construct  $D^*$ . One glaring limitation of our result is that traders need to “reserve” liquidity for an event regardless of whether it is realized or not. This can be partially ameliorated by combining mutually disjoint events as follows.

**Decomposition 1** *Let  $D^*$  be expressed as:*

$$D^*(\omega) = \sum_k c_k^* H_k$$

Where  $H_k$  has coarseness  $\mathcal{H}_k \subseteq \mathcal{G}^*$  a family of disjoint events and  $\|H_k\|_1 \leq 1$ . Note that  $\mathcal{H}_k$  could still a singleton event.

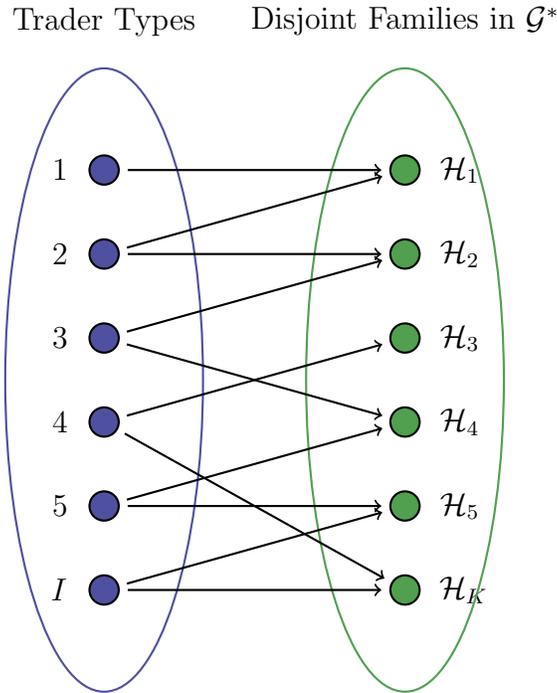


Figure 2: Bipartite flow matching problem with *disjoint families* in  $\mathcal{G}^*$ .

**Theorem 1 (Sustaining Efficiency)** *An aggregate demand function  $D^*$  with coarseness  $\mathcal{G}^* = \cup_i \mathcal{G}_i$  that leads to equal returns across assets (conditional on  $\mathcal{G}^*$ ) can be*

supported in equilibrium if for all subsets  $\mathcal{K} \subseteq \{1, \dots, K\}$ :

$$\sum_{k \in \mathcal{K}} |c_k^*| \leq \sum_{\{i : \exists k \in \mathcal{K} \text{ s.t. } \mathcal{H}_k \subseteq \mathcal{G}_i\}} l_i$$

Where  $c_k^*$  corresponds to the coefficient of  $H_k$  in Decomposition 1 of  $D^*$ .

**Proof.** Hall’s marriage theorem and the triangle inequality (see Appendix B). ■

Much like the welfare theorems, the point here is not to say that markets are informationally efficient. Rather, it is to provide a framework for understanding why they may be more or less efficient. To the extent our sufficient conditions do not hold, markets might not be able to aggregate private analyses.

## 5 Variably Diffuse Information

Defining analysis as a form of information helps rectify our intuitions about how difficult it is for markets to aggregate public data. Our EMH\* and aggregation results help flesh out the concept of analysis, but they do not explain the need for technical analysts in particular. The next question is, how do patterns arise that seem economically spurious and cannot be well accounted for by value investors or liquidity providers?

In standard linear rational expectations equilibrium models (Grossman and Stiglitz 1980; Hellwig 1980; Kyle 1985, 1989; Rostek and Weretka 2012; Goldstein and Yang 2015; Lambert, Ostrovsky, and Panov 2018), value investors mainly counteract additive demand (and supply) shocks known as noise trading. As the number of value investors grows large, prices become fully revealing in many of these models. However, there are many other potential sources of noise that value investors may not be able to aggregate away, e.g. the winner’s curse in Miller (1977).

This section considers homogenous traders who draw signals for the value of an asset from some aggregate distribution with *variable moments*. While hard to economically interpret, this variation in how information is spread out across traders will interact with demand curvature and generate pricing errors. To the extent traders

have not already accounted for such errors, there will be seemingly spurious patterns between related variables. For example, short interest and analyst ratings relate to distributional moments, and macroeconomic variables affect demand curvature through liquidity and risk preferences.

Using as simple of a model as possible, we illustrate many different ways that information could be variably diffuse and the resulting price behaviors. Appendix D grounds this model in the standard rational expectations literature with simplifying constraints on trading behavior. For exposition, we present the simplified model here on its own.

## 5.1 Toy Model Setup

Consider a continuum of homogenous traders buying one divisible asset. Each trader receives a private signal  $s \sim N(r, \sigma_s^2)$  for the log rate of return  $r$  and then chooses demand  $D(s)$  for the asset in terms of the numeraire. Notably, the input to the trader's problem here is unidimensional. Rather than observing both the price and an estimate for asset value, traders divide the two and get a single estimate  $s$  for log return  $r$ .

Supply of the asset in terms of the numeraire is simply its price, which is inversely related to log return  $r$ . Given some demand shock  $\epsilon \sim N(0, \sigma_\epsilon^2)$ , an equilibrium is characterized by demand  $D^*$  and log return  $r^* = R^*(\epsilon)$  satisfying market clearing:

$$w(\epsilon, r^*) \equiv \underbrace{\log \left( \mathbb{E}_s [D^*(s) | r^*] \right)}_{\text{Log Aggregate Demand}} + \epsilon + r^* = 0 \quad (1)$$

Intuitively, higher demand leads to higher prices and lower rates of return. While the market clearing condition here is taken as given, it derives from a rational expectations equilibrium model in Appendix D.

Lastly, the equilibrium demand function must be a best-reply:

$$D^*(s) \equiv \arg \max_{x \subseteq X} \mathbb{E}_r [u(x, r) | s, R^*] \quad (2)$$

Where  $u$  is the traders ex-post utility given demand  $x$  (within budget constraint  $X$ ) and rate of return  $r$ .

Table 3: Functions.

$R(\epsilon)$	Log return
$D(s)$	Demand

## 5.2 General Equilibrium Properties

This unidimensional model has the following well-behaved equilibrium properties. Since  $s | r$  satisfies the monotone likelihood ratio property (MLRP) due to normality, demand is increasing for any return function (assuming  $u_{xr} > 0$ , single-crossing).

**Property 2**  $D^*(s | R)$  is strictly increasing in  $s$  for any nonconstant  $R(\epsilon)$ .

Because aggregate demand is increasing in any equilibrium, market clearing is more sensitive to returns than demand shocks,  $w_r \geq w_\epsilon = 1$ .

**Corollary 1** Any equilibrium  $R^*(\epsilon)$  is strictly decreasing in  $\epsilon$ .

**Corollary 2** The distribution of  $R^*(\epsilon)$  is a weak contraction of  $\epsilon$ .

For a large class of distributions (MLRP) and utility functions (single-crossing), our model only admits monotone equilibria that are analytically useful. Corollary 2 adds that returns will always be less noisy than the underlying demand shocks.

## 5.3 The Winner's Curse Extended

The logic of the winner's curse in Miller (1977) is that, when there are many traders who cannot short trade, then unaccountedly disperse signals lead to high prices and subpar returns. We generalize it here as the interaction between variance and demand curvature.

**Observation 1** If equilibrium demand  $D^*$  is convex/concave, then unaccountedly high signal variance  $\sigma_s^2$  leads to greater/lower demand and therefore lower/greater returns.

Estimating such a latent interaction is difficult especially if traders are not even aware of it and only focus on fundamentals. Our point is that a whole host of economic

variables could relate to either this signal variance or demand curvature, and the latent as well as spurious nature of these interactions give a comparative advantage to general pattern recognition over structural economic models. This thereby creates a potential niche for technical analysts who profit off these shorter fluctuations rather than longer term fundamentals.

Another variable that could have unexpectedly high variation are the demand shocks. How are returns effected (in expectation) during periods of unexpectedly high demand volatility? The answer is somewhat analogous but notably depends on the curvature of aggregate demand instead of individual demand.

**Observation 2** *If equilibrium log aggregate demand  $\log(\mathbb{E}_s[D^*(s) | r^*])$  is convex/concave with respect to return  $r^*$ , then unaccountedly high demand shock variance  $\sigma_\epsilon^2$  leads to lower/greater returns in expectation.*

This rigorously follows from the implicit function theorem. Intuitively, if log aggregate demand were convex, then large negative demand shocks have diminishing effects on returns while large positive demand shocks have increasing effects.

The complex interactions in this simple unidimensional model belie how difficult it is for markets to aggregate variably diffuse information and the need for non-traditional traders. Traders caring in part about log returns means curvature is unavoidable here.

**Observation 3** *Individual demand  $D^*(s)$  and log aggregate demand  $\log(\mathbb{E}_s[D^*(s) | r^*])$  cannot both be linear.*

While linear rational expectations equilibrium models are analytically tractable, given liquidity constraints and the variety of risk preferences that traders have, there is little reason to expect linear demand in practice.

However, without linearity, problems aggregating variably diffuse information arise rather generally beyond variances.

**Observation 4** *Variation in moments of  $\sigma_s^2$  and  $\sigma_\epsilon^2$  interact with the corresponding Taylor series orders of  $D^*(s)$  and  $\log(\mathbb{E}_s[D^*(s) | r^*])$  respectively to generate pricing errors.*

## 5.4 Cutoff Demand Example

For a concrete example, we suppose investors are risk neutral (over log returns) but liquidity constrained. Then, from property 2, individual demand is without loss of generality the cutoff rule:

$$D^*(s | R) = \begin{cases} L & s \geq s^* \\ 0 & s < s^* \end{cases}$$

Aggregate demand, where  $\Phi$  is the normal cdf, is:

$$\mathbb{E}_s [D(s) | \epsilon] = L \cdot \Phi(-z^*(\epsilon))$$

$$z^*(\epsilon) \equiv \frac{s^* - R^*(\epsilon)}{\sigma_s}$$

And, market clearing becomes:

$$\log(\Phi(-z^*(\epsilon))) + \log(L) + \epsilon + R^*(\epsilon) = 0$$

We can understand price informativeness through the magnitude of the slope of  $R^*$ , which is given by the implicit function theorem as:

$$R_\epsilon^*(\epsilon) = \frac{-1}{H(z^*(\epsilon)) + 1}$$

Where  $H$  is the standard normal hazard function, which increases in  $z^*$ . The flatter  $R^*(\epsilon)$  is the more informative prices are.

**Property 3** *There is a unique equilibrium and  $z^*(\epsilon)$  strictly increases in liquidity  $L$ .*

Then,  $|R_\epsilon^*|$  is decreasing in liquidity and prices become more informative with liquidity.

**Corollary 3** *For  $L_1 < L_2$ ,  $R^*(\epsilon | L_1)$  is a weaker contraction of  $\epsilon$  than  $R^*(\epsilon | L_2)$ .*

This result follows generally for non-normal distributions if the hazard function  $H$  is increasing.

Because the normal hazard function grows arbitrarily large, we get that prices become arbitrarily informative with liquidity, or equivalently the measure of traders here, which aligns with the literature.

**Corollary 4**  $\lim_{L \rightarrow \infty} z^*(\epsilon | L) = \infty$ , and  $\lim_{L \rightarrow \infty} R^*(\epsilon | L) = \bar{r}$  the implied outside rate of return where  $u_x(0, \bar{r}) = 0$ .

However, note that in this example, log aggregate demand  $\log(\Phi(-z^*(\epsilon)))$  is a concave function of  $r$  (because the normal hazard function is increasing). Thus, it follows from Remark 2 that markets with unexpectedly high demand shock variance  $\sigma_\epsilon^2$  are underpriced and give greater returns.

**Observation 5** *In this cutoff demand example, assets give greater returns in expectation when the demand shock variance  $\sigma_\epsilon^2$  is unexpectedly high.*

## 6 Conclusion

The purpose of this paper is to reconcile classical intuitions about efficient markets with the prevalence of technical analysts in practice. Our first step is to define analysis as a form of information, thereby distinguishing individual pieces of information from their many interactions. This new information structure allows for bounded rationality models where analysis is costly, such as correlation neglect. It then becomes apparent how strong the EMH is by suggesting markets account for all the information contained in public data—even analyses no one has conducted.

While the EMH is an empirically useful null hypothesis, a more conceptually sound version is that markets aggregate all private analyses. The question then becomes why might markets be more or less informationally efficient? To this end, we develop sufficient conditions for sustaining efficient aggregate demand in equilibrium—giving a

framework for what could go wrong. These rudimentary results further serve as examples of how to do theoretical work with families of events as a new form of information.

Lastly, to reconcile theory and practice, we need to explain the unique informational role technical analysts play. Rather than directly modeling fundamentals or liquidity, they focus on broad statistical pattern recognition at short horizons. Section 5 illustrates how latent interactions between distributional variation and demand curvature can generate these broad short term patterns that seem economically spurious. This suggests technical analysts have a comparative advantage aggregating variably diffuse information, which gives empirical implications for what markets they are prevalent in.

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## A Review of $\sigma$ -Fields

A  $\sigma$ -field  $\mathcal{F}$  is any family of events, i.e.  $\mathcal{F} \subseteq 2^\Omega$  where  $\Omega$  is the sample space, satisfying the three properties that it:

1. Is closed under complements, if  $E \in \mathcal{F}$  then  $E^c \in \mathcal{F}$ .
2. Contains the sample space,  $\Omega \in \mathcal{F}$ . (This implies that  $\emptyset \in \mathcal{F}$  by closure under complements.)
3. Is closed under countable intersection, if  $E_1, E_2, \dots \in \mathcal{F}$  then  $\bigcap_{i=1}^{\infty} E_i \in \mathcal{F}$ . (This implies closure under countable unions because of closure under complements.)

A probability space is defined by a triplet  $(\Omega, \mathcal{F}, \mathbb{P})$  where  $\Omega$  is the sample space and  $\mathbb{P}$  is the probability measure defined over the  $\sigma$ -field  $\mathcal{F}$ . When the sample space is finite, then  $\mathcal{F}$  is often taken to be the “total  $\sigma$ -field,”  $\mathcal{F} = 2^\Omega$ . Much of the impetus for  $\sigma$ -fields is that complications arise when the sample space is larger, e.g.  $\Omega = \mathbb{R}$ , where  $\mathbb{P}$  cannot be defined over the power set without violating axioms of probability.

Information is always characterized by the sub  $\sigma$ -field it generates in the underlying probability space. Observing some family of events  $\{E_i\}_{i \in \mathcal{I}}$  corresponds to observing all events in the generated  $\sigma$ -field:

$$\sigma(\{E_i\}_{i \in \mathcal{I}}) \equiv \bigcap_{\substack{\hat{\mathcal{F}} \text{ is a } \sigma\text{-field} \\ \{E_i\}_{i \in \mathcal{I}} \subseteq \hat{\mathcal{F}}}} \hat{\mathcal{F}}$$

That is, the information someone has is the closure under the three  $\sigma$ -field properties of the events they observe. If someone observes a random variable, then they observe events corresponding to each possible value of that variable. The knowledge an information set describes is the  $\sigma$ -field generated by it.

Our paper makes no contribution to probability theory which describes what can be inferred from pieces of information. However, unlike probability, economics needs a broader definition of information that recognizes the cost of analyzing interactions.

We argue the way to do this is by letting any family  $\mathcal{G} \subseteq \mathcal{F}$  characterize knowledge. The following examples illustrate the usefulness of this family notion.

### A.1 Detailing the Separability Problem in Example 1

Consider the two random variables  $X_1$  and  $X_2$  in probability space  $(\Omega, 2^\Omega, \mathbb{P})$ .

		$X_2$		
	$\Omega$	0	1	2
	0	$\omega_1$	$\omega_2$	$\omega_3$
$X_1$	1	$\omega_4$	$\omega_5$	$\omega_6$

Knowing  $X_1$  corresponds to observing the two events:

$$E_1^0 = \{X_1 = 0\} = \{\omega_1, \omega_2, \omega_3\}$$

$$E_1^1 = \{X_1 = 1\} = \{\omega_4, \omega_5, \omega_6\}$$

And, knowing  $X_2$  corresponds to observing the three events:

$$E_2^0 = \{X_2 = 0\} = \{\omega_1, \omega_4\}$$

$$E_2^1 = \{X_2 = 1\} = \{\omega_2, \omega_5\}$$

$$E_2^2 = \{X_2 = 2\} = \{\omega_3, \omega_6\}$$

Any event  $W_i = \{\omega_i\}$  can be written as the intersection of  $E_1^0$  or  $E_1^1$  with one of  $E_2^0$ ,  $E_2^1$ , or  $E_2^2$ . For example,  $W_1 = \{\omega_1\} = E_1^0 \cap E_2^0$ , which is the event that both  $X_1$  and  $X_2$  are zero. Since any singleton event can be generated this way, and  $\sigma$ -fields are closed under countable union:

$$\sigma(E_1^0, E_1^1, E_2^0, E_2^1, E_2^2) = 2^\Omega$$

That is, there is no way in the given probability space to describe separately knowing  $X_1$  and  $X_2$  but not their interactions. Anyone who observes the five events  $E_1^0, E_1^1, E_2^0,$

$E_1^2$ , and  $E_2^2$  must necessarily also know how to condition on each of the six possible realizations in the sample space  $\Omega$ .

Notably, sufficient statistics cannot solve this separability problem since they still must generate a sub  $\sigma$ -field of  $2^\Omega$ , which cannot be just the five aforementioned events. One solution is to define a larger information structure, e.g. priors or a global game, whereby the interaction between the two variables is meaningless. Even though their interaction can be conditioned on, the two variables are effectively separable. However, this does not lend itself to an efficient market hypothesis that directly specifies what data is and is not accounted for.

By using families instead of  $\sigma$ -fields to define information, we are able to directly specify notions such as separability in terms of conditioning on data. Traders are able to separately know  $X_1$  and  $X_2$  without their interactions. While the difference between conditioning on five events versus each of the six realizations in the sample space is relatively small, and analyzing their interactions would be trivial, the nuance becomes critical in large data sets.

## B Sustaining Efficient Demand

This appendix proves Theorem 1 and therefore the special case of Result 1.

**Proof.** By assumption, Hall's marriage theorem applies to the bipartite flow matching problem in Figure 2. Hence, there exists a flow  $\{f_{k,i}\}_{k=1,i=1}^{K,I}$  such that:

$$\sum_{i=1}^I f_{k,i} = c_k^* \quad \text{and} \quad \sum_{k=1}^K f_{k,i} \leq l_i$$

Let all traders of information type  $i$  have the demand function:

$$D_i = \frac{1}{l_i} \sum_{k=1}^K f_{k,i} H_k$$

By construction, each trader's liquidity constraint is satisfied:

$$\begin{aligned}\|D_i(\omega)\|_1 &= \left\| \frac{1}{l_i} \sum_{k=1}^K f_{k,i} H_k \right\|_1 \\ &\leq \frac{1}{l_i} \sum_{k=1}^K f_{k,i} \|H_k\|_1 \\ &\leq 1\end{aligned}$$

And, the traders together aggregate to the efficient demand function  $D^*$ .

$$\begin{aligned}\sum_{i=1}^I l_i D_i &= \sum_{i=1}^I \sum_{k=1}^K f_{k,i} H_k \\ &= \sum_{k=1}^K c_k^* H_k \\ &= D^*\end{aligned}$$

Since  $D^*$  results in assets having equal returns conditional on  $\mathcal{G}^* = \cup_i \mathcal{G}_i$ , small risk neutral traders are indifferent between all of their possible demand functions. ■

## C Variably Diffuse Information

The point of this section is to

### C.1 Setup

One divisible asset of common value  $v$  is traded on a financial market with a continuum of investors. The total measure of investors is  $m$  and each observes a private signal  $\hat{s}$ . Investors can buy the asset at an endogenously determined price  $p$  and then realize a log rate of return  $r = \log(v/p)$ . The ex-post utility from investing  $x$  dollars is given by a vNM utility function  $u(x, r)$  with basic regularity conditions.

The two exogenous variables in the model are value  $v$  and *mismatch*  $\epsilon \equiv \log(m/v)$ :

$$v \sim \log N(\mu_v, \sigma_v^2)$$

Table 4: Variables.

$v$	Value
$p$	Price
$r$	Log return
$m$	Measure of investors
$\hat{s}$	Private signal
$x$	Investment
$\epsilon$	Mismatch

$$\epsilon \equiv \log(m/v) \sim N(\mu_\epsilon, \sigma_\epsilon^2)$$

These variables are independent, which is an economic assumption that investors match with assets in proportion to value.

The distribution of private signals is centered around value:

$$\hat{s}|v \sim \log N(\log(v), \sigma_s^2)$$

Since investors lie on a continuum, this is the realized aggregate distribution of all private signals. Investors are otherwise homogenous and so their behavior is modeled with a representative agent. Everything about the model is common knowledge.

For a given demand function  $\hat{D}(\hat{s}, p)$ , the market clears if:

$$\begin{aligned} m \cdot \mathbb{E}_{\hat{s}} \left[ \hat{D}(\hat{s}, p) \mid v \right] &= p \\ \Leftrightarrow \mathbb{E}_{\hat{s}} \left[ \hat{D}(\hat{s}, p) \mid v \right] &= \frac{p}{v} \cdot \frac{v}{m} \\ \Leftrightarrow \log \left( \mathbb{E}_{\hat{s}} \left[ \hat{D}(\hat{s}, p) \mid v \right] \right) + r + \epsilon &= 0 \end{aligned}$$

From this equation we define an equilibrium as follows.

**Definition 4** *An equilibrium is characterized by a log return function  $\hat{R}^*(v, \epsilon)$  that clears the market for all  $v$  and  $\epsilon$ :*

$$\underbrace{\log \left( \mathbb{E}_{\hat{s}} \left[ \hat{D}^* \left( \hat{s}, p \mid \hat{R}^* \right) \mid v \right] \right)}_{\text{Log Aggregate Demand}} + \hat{R}^*(v, \epsilon) + \epsilon = 0 \quad (3)$$

Where  $p = ve^{-\hat{R}^*(v,\epsilon)}$ , and the expected utility maximizing level of investment is:

$$\hat{D}^* (\hat{s}, p \mid \hat{R}) \equiv \arg \max_{x \in \mathbb{R}} \mathbb{E}_r [u(x, r) \mid \hat{s}, p, \hat{R}] \quad (4)$$

Table 5: Functions.

$\hat{R}(v, \epsilon)$	Price
$\hat{D}(\hat{s}, p)$	Demand

## C.2 Scale Invariance

What does the equilibrium demand function look like? Intuitively, the most relevant piece of information to investors is the ratio of their signal to the price:

$$s \equiv \log(\hat{s}/p)$$

Along this line of thought, Section C.3 supposes investors can only condition on their normalized signal  $s$ , thus demand is given by  $D(s)$ . Then, Claim 1 shows market clearing returns can be selected independent of value as  $R(\epsilon)$ . This leads to a simple definition of equilibria where price and demand are univariate functions.

Using this simplified model, Section 5.2 proves various equilibrium properties, e.g.  $R^*(\epsilon)$  is a decreasing contraction. These properties then let us justify conditioning on normalized signals as rational inattention as well as argue it approximates equilibria of the general model.

## C.3 Equilibria

If investors only condition on  $s$ , the following claim proves that: under any demand function for which there exist market clearing returns, there exist market clearing returns that do not depend on value.

**Claim 1** *Given demand  $D(s)$ , if there exist returns  $\hat{R}(v, \epsilon)$  that clear the market, then there exist returns  $R(\epsilon)$  that clear the market as well.*

**Proof.** Fix  $\hat{v}$  and let  $R(\epsilon) \equiv R(\hat{v}, \epsilon)$ , then  $R(\epsilon)$  clears the market for  $\hat{v}$  and all  $\epsilon$  by construction. Now, normalized signals  $s = \log(\hat{s}/p)$ , where  $p = v e^{-R(\epsilon)}$ , have the distribution:

$$s = \log(\hat{s}/v) + R(\epsilon) \sim N(R(\epsilon), \sigma_s^2)$$

The market clearing equation is then constant in  $v$ :

$$\Leftrightarrow \log(\mathbb{E}_s [D(s) | \epsilon]) + R(\epsilon) + \epsilon = 0$$

Hence,  $R(\epsilon)$  clears the market for all  $v$  and  $\epsilon$ . ■

If investors only condition on  $s$ , and markets select clearing returns that do not depend value, then an equilibrium is defined as follows.

**Definition 5** A “scale invariant equilibrium” is characterized by a log return function  $R^*(\epsilon)$  that clears the market for all mismatch  $\epsilon$ :

$$\log(\mathbb{E}_s [D^*(s | R^*) | \epsilon]) + R^*(\epsilon) + \epsilon = 0 \tag{5}$$

Where investors maximize their expected utility:

$$D^*(s | R) \equiv \arg \max_x \mathbb{E}_r [u(x, r) | s, R] \tag{6}$$

With the signal distribution  $s \sim N(R(\epsilon), \sigma_s^2)$ .

Table 6: Scale invariant functions.

$R(\epsilon)$	Log return
$D(s)$	Demand

Since  $\log(p) = \log(v) + R^*(\epsilon)$  where  $v$  and  $R^*(\epsilon)$  are independent, Corollary 2 lets us bound how informative the price level is about returns.

**Corollary 5**  $Corr(\log(p), R^*(\epsilon)) = \sigma_r / \sqrt{\sigma_v^2 + \sigma_r^2} < \sigma_\epsilon / \sqrt{\sigma_v^2 + \sigma_\epsilon^2}$ .

The price level only conveys information about returns through the prior over values. Thus, it is arbitrarily uninformative for large  $\sigma_v/\sigma_r > \sigma_v/\sigma_\epsilon$ . This holds for both informed and uninformed investors since the signal distribution is independent of price level (conditional on returns).

Therefore, scale invariant equilibria can be supported by *rational inattention* to price level if the prior over values is sufficiently flat. As this prior represents the beliefs of investors before observing a signal or the price, it can be interpreted as the distribution of all values over some relevant broad class of assets. Thus, we argue flat priors are plausible, and that it would be difficult for investors to properly estimate and generally account for  $\sigma_v$ .

To the extent investors do account for  $\sigma_v$ , we expect the price level to become even less informative about returns. Thus, an equilibrium in this simplified model ought to closely approximate that in the general model. (Note that we do not expect to see the sort of unravelling that can happen with priors in coordination games.)